# The evolutionary dynamics of sexual dimorphism 

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## Which type of Ecological Polymorphism?



Purple-throated Carib (Eulampis jugularis)


Heliconia caribaea


Heliconia bihai

Species? Sexes?

Within-sexes?
Dominance-Recessivity?

## Overview

Sexual dimorphism or genetic polymorphism?

Sexual selection and sexual dimorphism

Fish

Character displacement and
Sexual dimorphism/Speciation
Slatkin (1984)

Character displacement can make species or sexes diverge, and the models look almost the same

Doebeli and Dieckmann $(1999,2003)$

The evolution of character displacement can lead to speciation

## Speciation and sexual selection

Sch/uter (2000)
"The addition of sexual selection to theories of speciation represents perhaps the single greatest theoretical advance in the area of speciation in adaptive radiation."

Shine (1989)

Ecological differences between the sexes: cause or consequence of sexual dimorphism?

## "Emergence" of Genetic Variation

Evolutionary Branching Point


## Sexual dimorphism and Mate Choice Evolution

If sexual dimorphism can evolve, can we still get evolutionary branching?

When mating is non-random, does this affect the evolution of sexual dimorphism?

When mating is non-random, do we still get evolutionary branching?


Variation in female preference that acts as a resource distribution for males

Branching in "mating" loci

Male mate choice

Local mating pools

## Two Sex Model

## Autosomal genes for sex phenotypes sex phenotypes are a reaction norm with two states

$$
\boldsymbol{N}_{t+1}=A_{t} \cdot N_{t}
$$

$\boldsymbol{A}_{t}=\boldsymbol{E}_{t} \cdot \boldsymbol{M}_{t}$

Resident population density

$$
N=\left(N_{\text {females }}, N_{\text {males }}\right)
$$

Mating Matrix M
Ecological Feedback E

## Two Sex Model

Mating Process

## Ecology

$\boldsymbol{M}_{t}=\frac{r}{4}\left(\begin{array}{ll}m_{1, t} & m_{2, t} \\ m_{1, t} & m_{2, t}\end{array}\right)$
$\boldsymbol{E}_{t}=\left(\begin{array}{cc}E_{f, t} & 0 \\ 0 & E_{m, t}\end{array}\right)$

Female dominant mating system

$$
m_{1}=c_{t}
$$

$$
m_{2}=c_{t} N_{f, t} / N_{m, t}
$$

## Two Sex Model

Mutant population dynamics

$$
\begin{aligned}
& N_{t+1}^{\prime}=A^{\prime}\left(z^{\prime}, z\right) \cdot N_{t}^{\prime} \\
& \boldsymbol{E}^{\prime}=\left(\begin{array}{cc}
E\left(z_{f}^{\prime}, z\right) & 0 \\
0 & E\left(z_{m}^{\prime}, z\right)
\end{array}\right)
\end{aligned}
$$

$$
\boldsymbol{M}^{\prime}=\frac{r}{4}\left(\begin{array}{ll}
c\left(z_{f}^{\prime}, z_{m}, z_{m}\right) & c\left(z_{f}, z_{m}^{\prime}, z_{m}\right) \frac{\hat{N}_{f}(z)}{\hat{N}_{m}(z)} \\
c\left(z_{f}\right) z_{m} & z_{m} \\
z_{m} & c\left(z_{f}, z_{m}^{\prime}, z_{m}\right) \frac{\hat{N}_{f}(z)}{\hat{N}_{m}(z)}
\end{array}\right)
$$

female that chooses chosen male

Trait vector

$$
z=\left(z_{f}, z_{m}\right)
$$

- Female traits
- Male traits
- prime denotes mutant


## Two Sex Model

## Invasion Fitness

$$
\lambda\left(z^{\prime}, z\right)=\frac{1}{2} \frac{c\left(z_{f}^{\prime}, z_{m}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}+\frac{1}{2} \frac{c\left(z_{f}, z_{m}^{\prime}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}
$$

When mate choice has no cost for females, one can set invasion fitness to

$$
\lambda_{\text {nocost }}\left(z^{\prime}, z\right)=\frac{1}{2} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}+\frac{1}{2} \frac{c\left(z_{f}, z_{m}^{\prime}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}
$$

Without sexual selection:

$$
\lambda_{\text {noss }}\left(z^{\prime}, z\right)=\frac{1}{2} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}+\frac{1}{2} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}
$$

## Two Sex Model

Canonical equation of evolutionary response

$$
\frac{d}{d t} z=\boldsymbol{M} \beta(z)
$$

$M$ is the mutational variance-covariance matrix scaled

Invasion Fitness Gradient $\beta$

$$
\beta(z)=\nabla^{\prime} \lambda(z, z)=\binom{\frac{\partial}{\partial z_{f}^{\prime}} \lambda\left(z^{\prime}, z\right)_{z^{\prime}=z}}{\frac{\partial}{\partial z_{m}^{\prime}} \lambda\left(z^{\prime}, z\right)_{z^{\prime}=z}}
$$

## Sexes or Species?

- Potential End Points of Evolution $z^{*}: \beta\left(z^{*}\right)=0$
- Invasibility: Eigenvalues of Hessian $H\left(z^{*}\right)$

- Convergence Stability or Reachability:

Eigenvalues of Jacobian $J\left(z^{*}\right)$

$$
J\left(z^{*}\right)=H\left(z^{*}\right)+Q\left(z^{*}\right)
$$

$$
\boldsymbol{H}\left(z^{*}\right)=D_{11} \lambda\left(z^{*}, z^{*}\right)
$$

$$
Q\left(z^{*}\right)=D_{12} \lambda\left(z^{*}, z^{*}\right)
$$

## Random Mating

evolutionary stops have

$$
\begin{aligned}
& \left.\frac{\partial}{\partial z_{f}^{\prime}} E\left(z_{f}^{\prime}, z\right)\right|_{z^{\prime}=z}=0 \\
& \left.\frac{\partial}{\partial z_{m}^{\prime}} E\left(z_{f}^{\prime}, z\right)\right|_{z^{\prime}=z}=0
\end{aligned}
$$

The ecological feedback is maximized

Male and female phenotypes can be equal or different

## Random Mating

evolutionary stops with equal phenotypes in females and males have

$$
\begin{aligned}
& D_{1} E\left(z^{*}, z^{*}\right)=0 \\
& D_{11} E\left(z^{*}, z^{*}\right)=h
\end{aligned} \quad D_{12} E\left(z^{*}, z^{*}\right)=\binom{q_{1}}{q_{2}}
$$

$$
\boldsymbol{H}\left(z^{*}\right)=\frac{1}{2}\left(\begin{array}{ll}
h & 0 \\
0 & h
\end{array}\right)
$$

$$
\boldsymbol{J}\left(z^{*}\right)=\frac{1}{2}\left(\begin{array}{cc}
q_{1}+h & q_{2} \\
q_{1} & q_{2}+h
\end{array}\right)
$$

## CSS, convergence stable, non-invadable

No sex differentiation

Sexual dimorphism can evolve

Stabilizing Selection


## Evolutionary Branching Points

No sex differentiation

Sexual dimorphism can evolve


## Disruptive Selection


trait value
females

## Dimorphic Sexes or Evolutionary Branching?

## Branching Point $\rightarrow$ Saddle Point

Srait value
males
Speciation will
usually occur
after passing
here

## What can save branching?

## "Secondary evolutionary branching"

Asymmetric competition example Large has an advantage over small

$$
E\left(z_{i}^{\prime}, z, \hat{N}\right)=1-\sum_{j=f, m} \frac{\alpha\left(z_{i}-z_{j}\right) N_{j}}{k\left(z_{i}\right)}
$$

$$
\alpha\left(z_{i}-z_{j}\right)=c\left(1-\frac{1}{1+v e^{-\gamma\left(z_{i}-z_{j}\right)}}\right)
$$

$$
k\left(z_{i}\right)=\frac{m}{\sqrt{2 \pi \sigma^{2}}} e^{-\frac{\left(z_{i}\right)^{2}}{2 \sigma^{2}}}
$$

## What can save speciation?

Asymmetric competition example

$$
C=1, v=1, \gamma=1, m=1, r=3
$$



$$
\gamma=0.75
$$



$$
\gamma=1.25
$$

## What can save branching?

Asymmetric competition example



Maximum

## What can save speciation?

Asymmetric competition example


## What can save branching?

A matching mechanism in mate choice


Branching restored!

## Sexual Dimorphism or What?

When allowing for sex differentiation,
the emergence of genes with major effects and
the possibility of sympatric speciation
will generally not occur as in the model without sex differentiation

Speciation can be saved by:

- Genetic constraints
- Secondary evolutionary branching



## Mate Choice

potential evolutionary stops $z^{*}$ have

$$
\begin{aligned}
& \left.\frac{\partial}{\partial z^{\prime}{ }_{f}} \frac{c\left(z_{f}^{\prime}, z_{m}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)}\right|_{z^{\prime}=z=z^{*}}+\left.\frac{\partial}{\partial z_{f}^{\prime}} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}\right|_{z^{\prime}=z=z^{*}}=0 \\
& \left.\frac{\partial}{\partial z^{\prime}{ }_{m}^{\prime}} \frac{c\left(z_{f}, z_{m}^{\prime}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)}\right|_{z^{\prime}=z=z^{*}}+\left.\frac{\partial}{\partial z_{m}^{\prime}} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}\right|_{z^{\prime}=z=z^{*}}=0
\end{aligned}
$$

There is a balance between sexual and natural selection
Male and female phenotypes will more often be different at an evolutionary stop

When there is no mating cost for females, mating traits only expressed in females are neutral (Fisherian Runaway)

## Mate Choice

Hessians have eigenvalues proportional to

$$
\begin{aligned}
& \left.\frac{\partial^{2}}{\partial z^{\prime}{ }_{f}{ }^{2}} \frac{c\left(z_{f}^{\prime}, z_{m}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)}\right|_{z^{\prime}=z=z^{*}}-2\left(\left.\frac{\partial}{\partial z_{f}^{\prime}} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}\right|_{z^{\prime}=z=z^{*}}\right)^{2}+\left.\frac{\partial^{2}}{\partial z_{f}^{\prime}{ }^{2}} \frac{E\left(z_{f}^{\prime}, z\right)}{E\left(z_{f}, z\right)}\right|_{z^{\prime}=z=z^{*}} \\
& \left.\frac{\partial^{2}}{\partial z_{m}^{\prime}{ }^{2}} \frac{c\left(z_{f}, z_{m}^{\prime}, z_{m}\right)}{c\left(z_{f}, z_{m}, z_{m}\right)}\right|_{z^{\prime}=z=z^{*}}-2\left(\left.\frac{\partial}{\partial z_{m}^{\prime}} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}\right|_{z^{\prime}=z=z^{*}}\right)^{2}+\left.\frac{\partial^{2}}{\partial z_{m}^{\prime}{ }^{2}} \frac{E\left(z_{m}^{\prime}, z\right)}{E\left(z_{m}, z\right)}\right|_{z^{\prime}=z=z z^{*}}
\end{aligned}
$$

If $c$ is concave, the pattern of invasibility is expected to tend more towards non-invasibility
$\rightarrow$ the likelihood of (secondary) branching should often be reduced

## The Evolutionary Ecology of Dominance - Recessivity

- Industrial melanism in the peppered moth (Haldane 1956)
- Batesian mimicry in butterflies (Clarke and Sheppard, 1960)
- wing dimorphism in insects (Roff and Fairbarn 1994)
- handedness in scale-eating cichlid fish (Hori 1993)


R

L

## Perissodus \& Handedness

Fitness Proxy: relative predation success

Predation succes =

P[attacking left]* ripping success left* Rarity advantage left
$+$
P[attacking right]* ripping success right * Rarity advantage right

## Perissodus \& Handedness

## Ripping Success

Gaussian with mean at Orientation - 0.5 (RF) or 0.5 (LF)
$\underbrace{\substack{\text { Right } \\ \text { flank }}}_{0}$

P[Attacking left]

$$
\frac{\exp (\text { orientation })}{1+\exp (\text { orientation })}
$$



Left Beak Orientation Right

## Perissodus \& Handedness


z Beak Orientation $z \in[-1,1]$, >> is more towards right

## Perissodus \& Handedness



