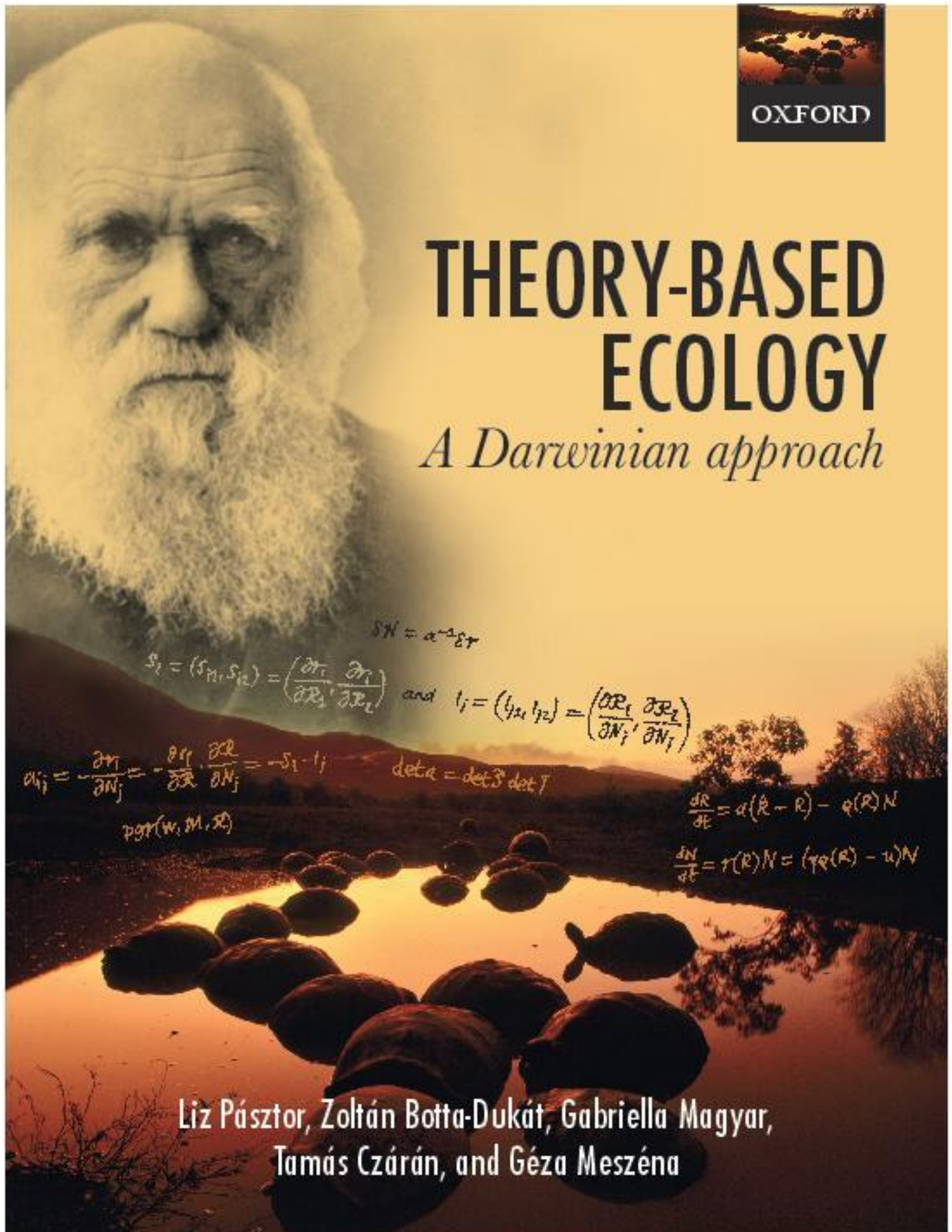


OXFORD

THEORY-BASED ECOLOGY

A Darwinian approach

Liz Pásztor, Zoltán Botta-Dukát, Gabriella Magyar,
Tamás Czárán, and Géza Meszéna

Theory in ecology?

What to do, if Real World is more complicated, than Lotka-Volterra?

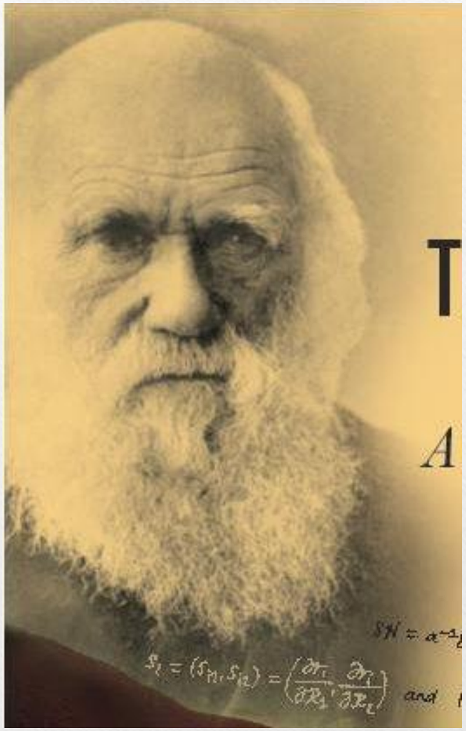
More complicated models will be even worse! Forget theory!

Plethora of sophisticated models, integrated verbally.

Theory, that integrates!

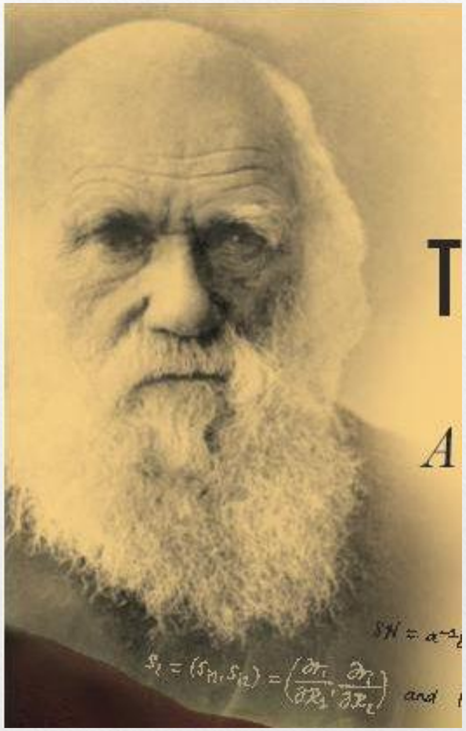
???

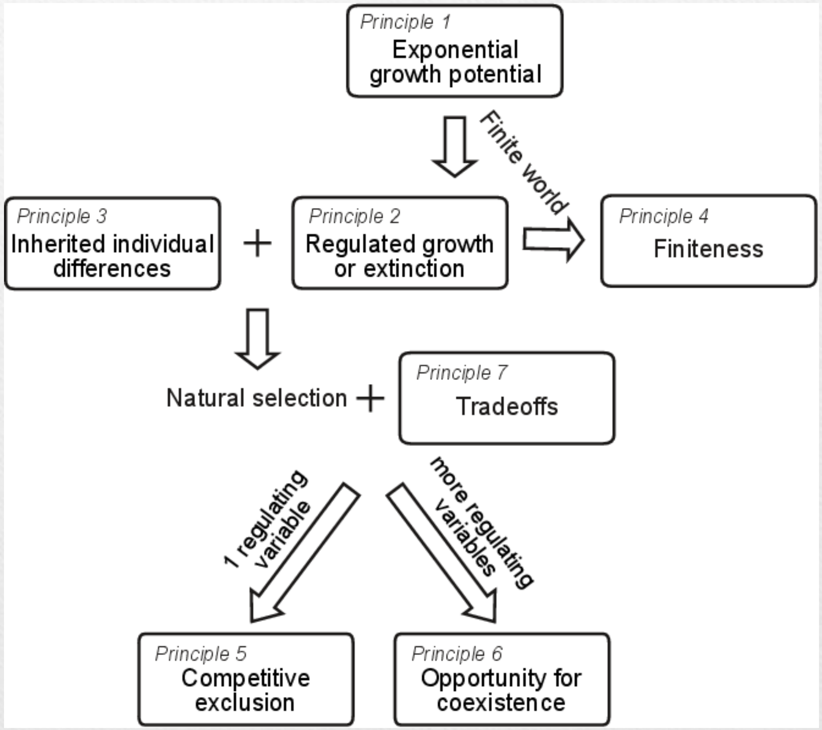
Darwinian principles



- **Exponential growth.**
- **Growth regulation.**
- **Inherited differences.**
- **Survival of the fittest.**
- **Constraints and tradeoffs.**
- **Robust coexistence.**
- **Finiteness of populations.**

Darwinian principles

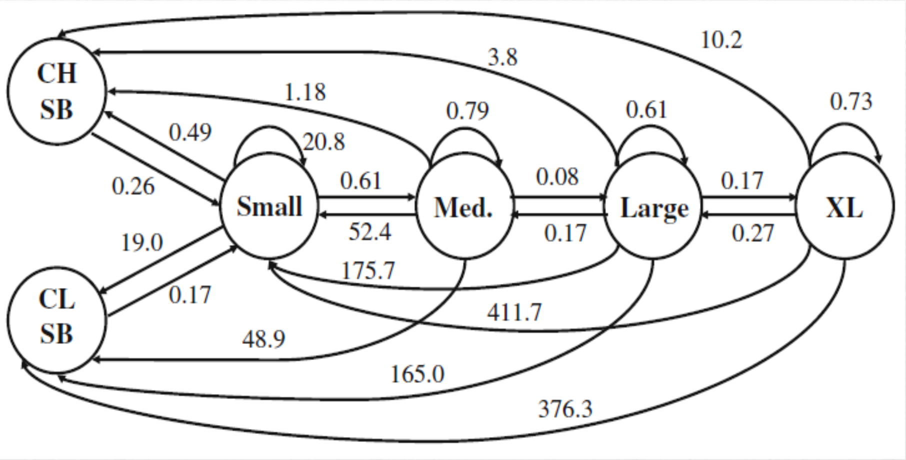




Sources of complexity

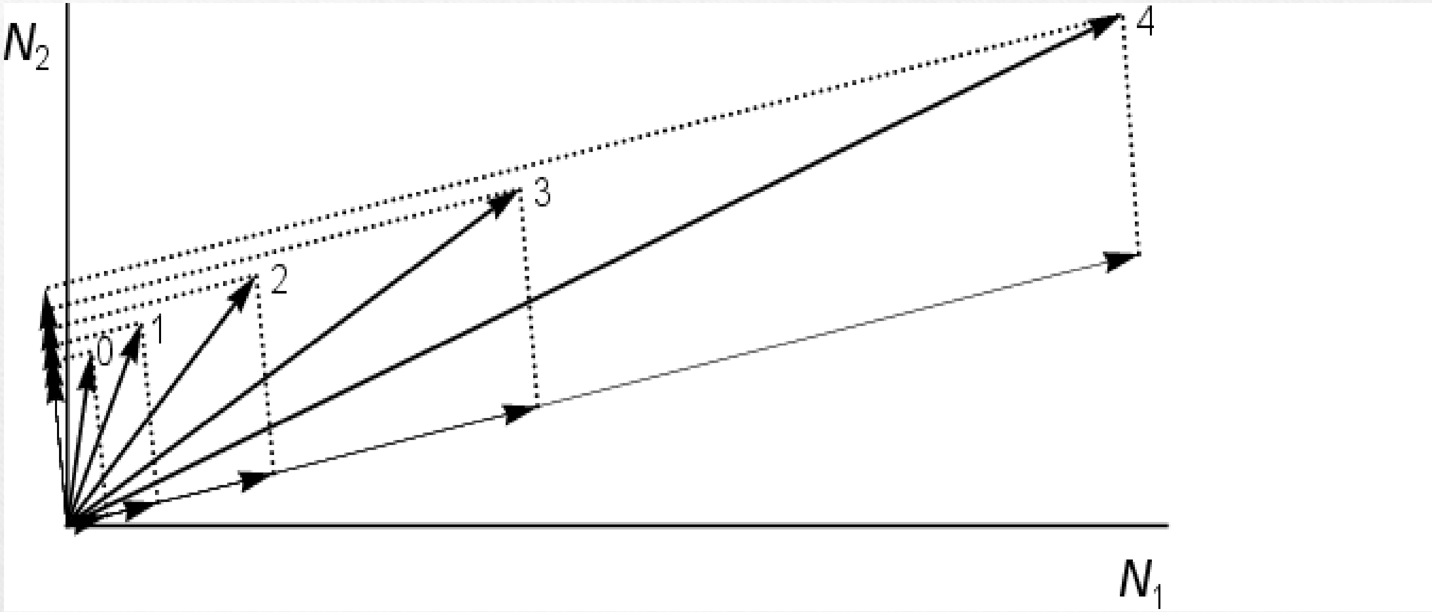
- Stochasticity of life histories
- *i*-states and structured populations
- Complexity of interactions between individuals
- Complexity of population dynamics
- Environments changing in space and time
- Complexity of communities
- Linking theory and data

Structured population: an example from the many



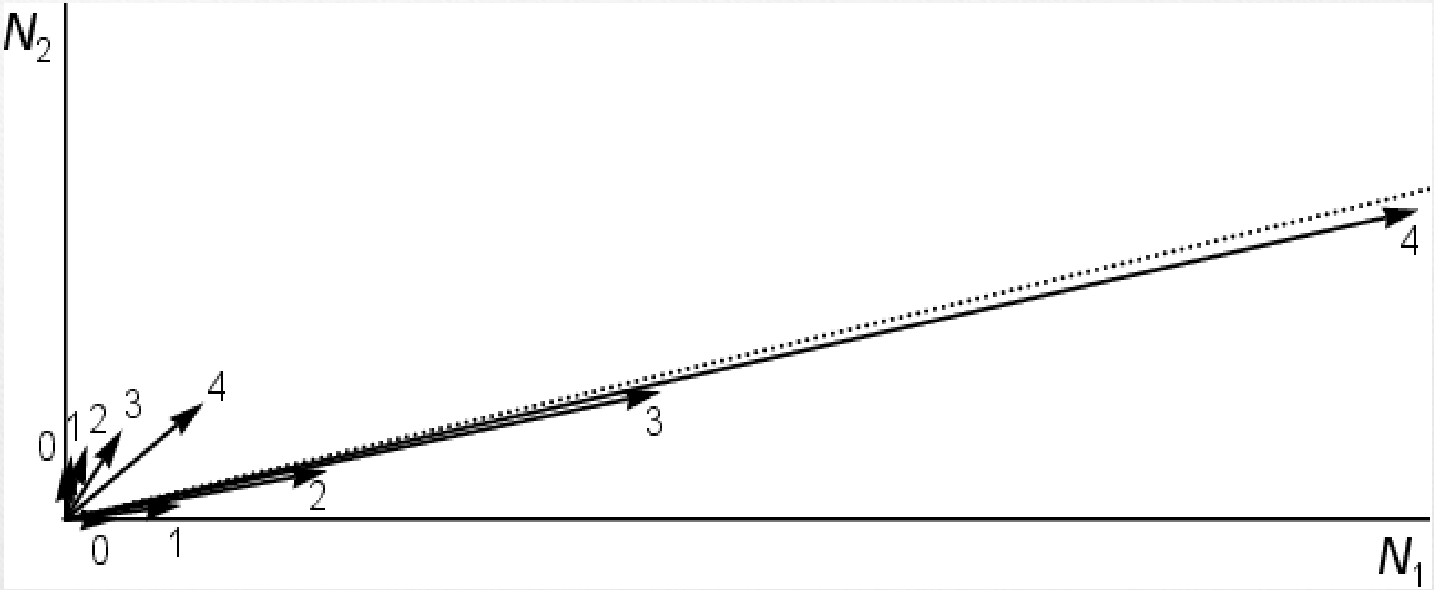
Lespedeza cuneata (Schutzenhofer et al. 2009)

Population structure: Matrix & eigenvector/value



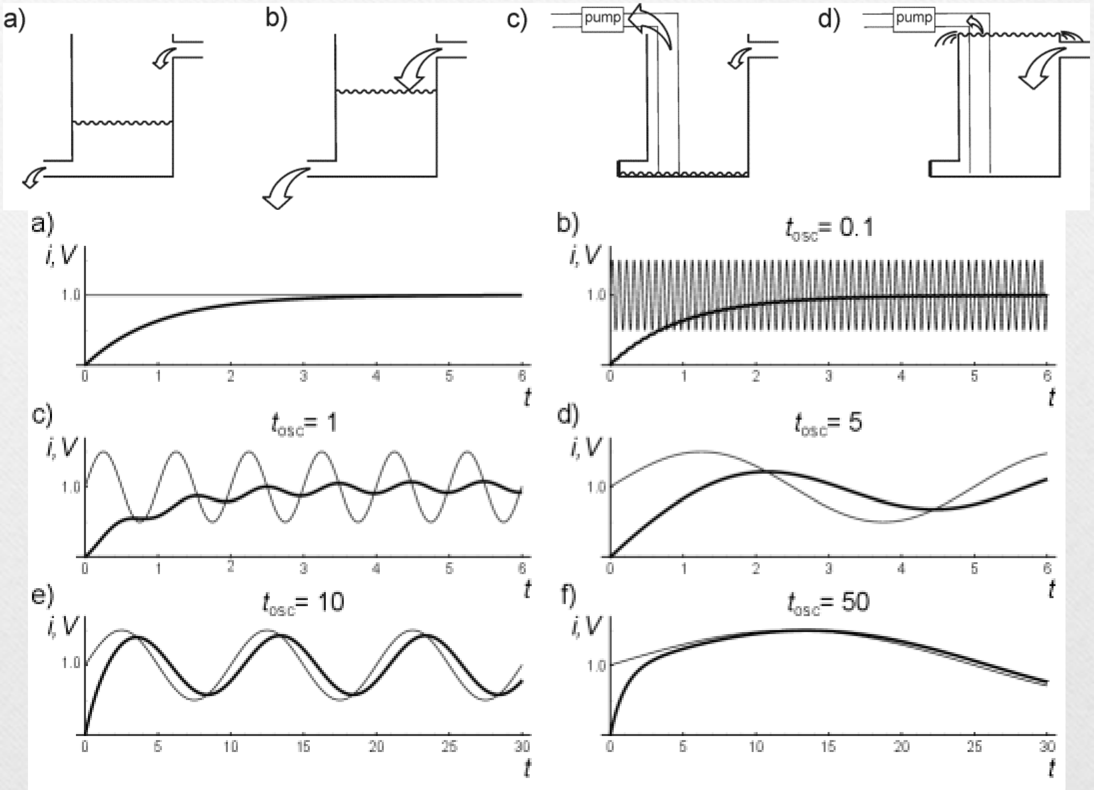
Unifying concept for *any* kinds of population structure!

Reproductive value

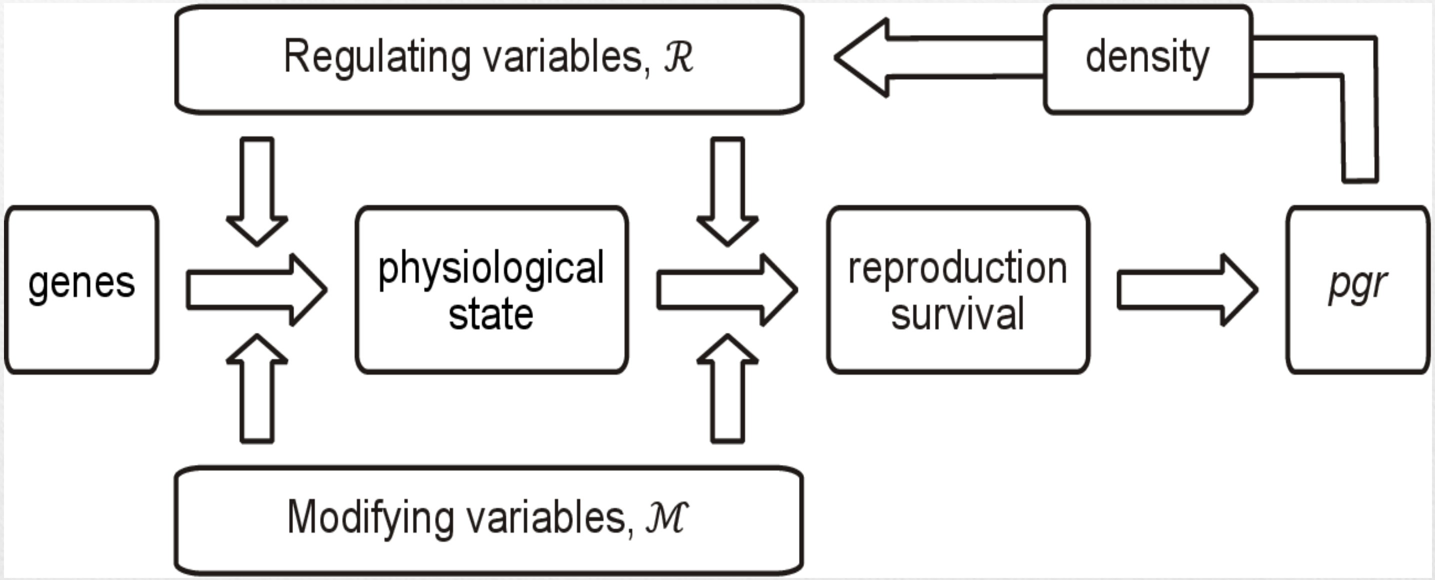


Advantage of starting in the better patch is retained indefinitely.

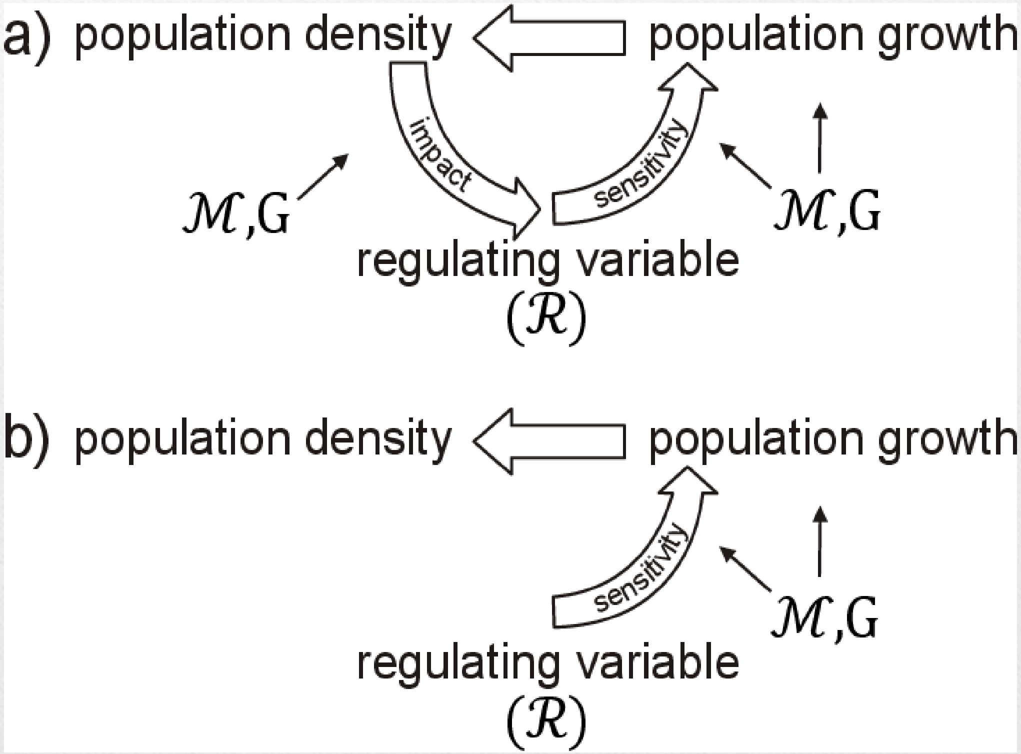
Negative feedback & time-scale separation



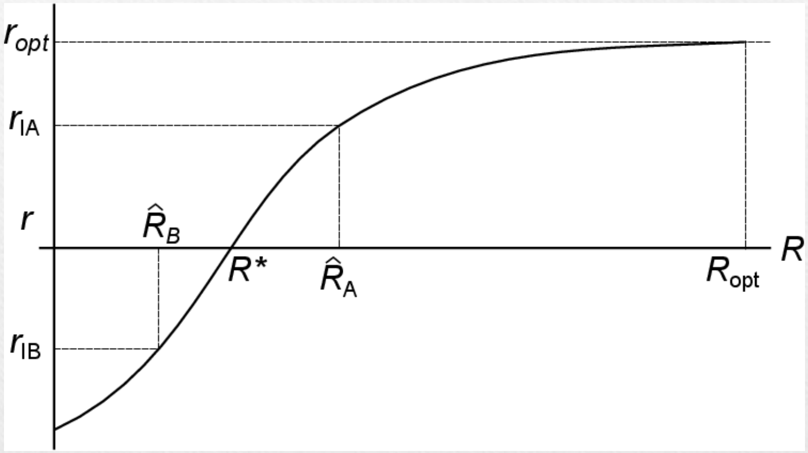
Chain of effects and the environmental feedback

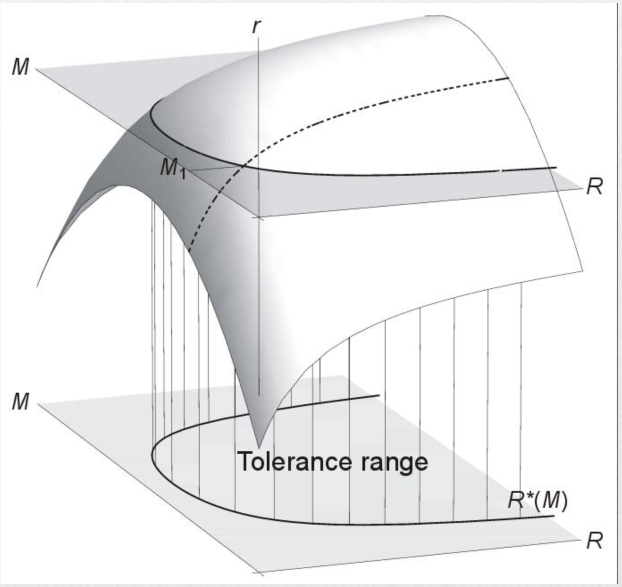


Open and closed loop of regulation

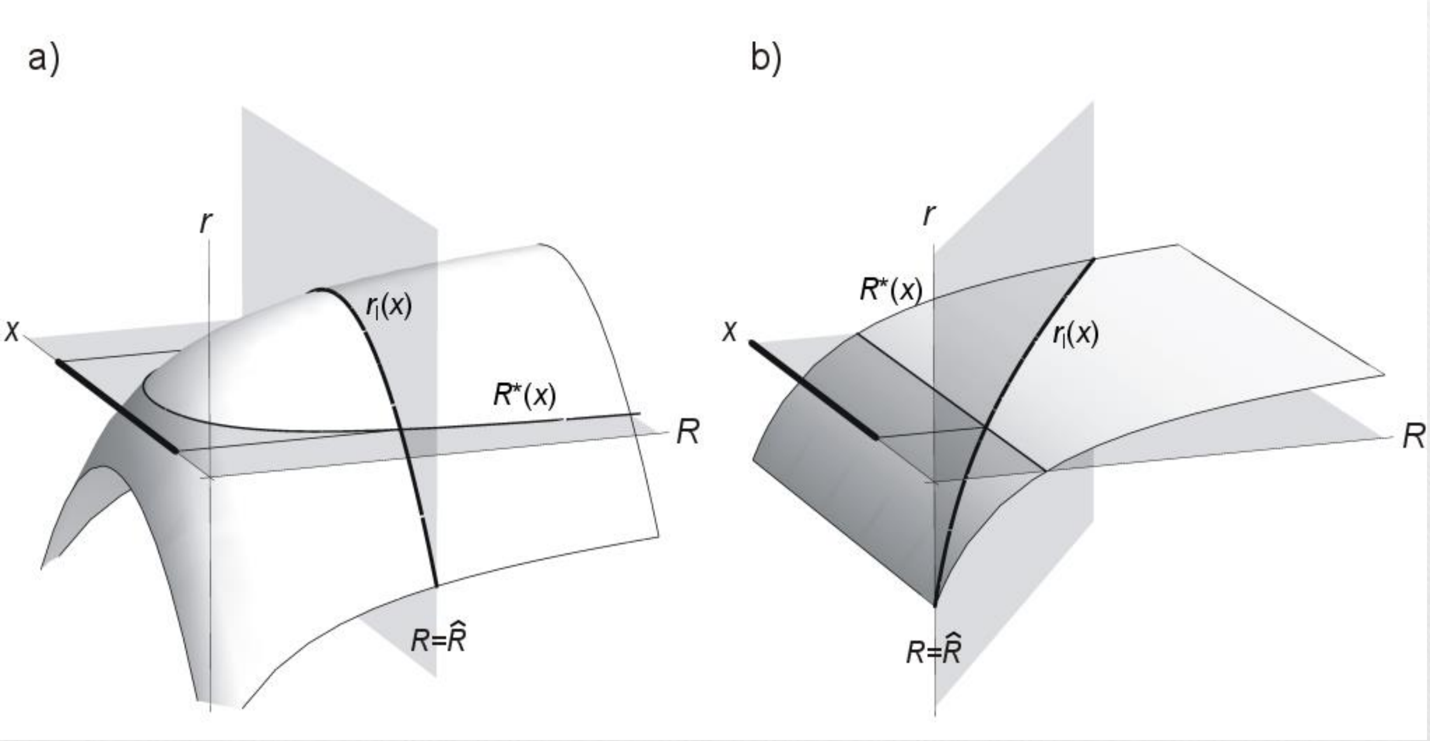


Response function of a population





Spatial distribution



Bottom-up control

Dynamics:

$$\frac{dR}{dt} = \alpha\left(\hat{R} - R\right) - \varrho(R)N$$

$$\frac{dN}{dt} = r(R)N = (\gamma\varrho(R) - u)N$$

Equilibrium:

$$N = \frac{\alpha}{\rho(R^*)}\left(\hat{R} - R^*\right)$$

$$R = R^*$$

Holling I, R is fast:

$$R = \bar{R}(N) = \frac{\alpha}{\alpha + \beta N}\hat{R}$$

$$I = \frac{d\bar{R}(N)}{dN} = -\frac{\alpha\beta}{(\alpha + \beta N)^2}\hat{R}$$

$$\bar{r}(N) = \frac{\alpha\beta\gamma}{\alpha + \beta N}\hat{R} - u$$

$$S = \frac{dr(R)}{dR} = \gamma\beta$$

$$a = -\frac{d\bar{r}(N)}{dN} = -S \cdot I = \frac{\alpha\beta^2\gamma}{(\alpha + \beta N)^2}\hat{R}$$

Top-down control

Dynamics:

$$\frac{dR}{dt} = \alpha\left(\hat{R} - R\right) - \beta^{\mathrm{N}} R N$$

$$\frac{dN}{dt} = \left(\gamma^{\mathrm{N}} \beta^{\mathrm{N}} R - u^{\mathrm{N}}\right) N - \beta^{\mathrm{P}} N P$$

$$\frac{dP}{dt} = \left(\gamma^{\mathrm{P}} \beta^{\mathrm{P}} N - u^{\mathrm{P}} - bP\right) P$$

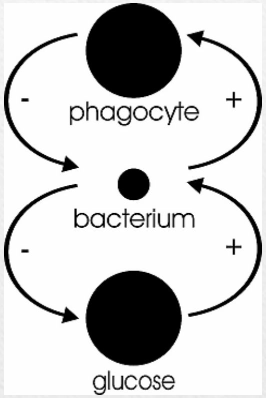
Regulation::

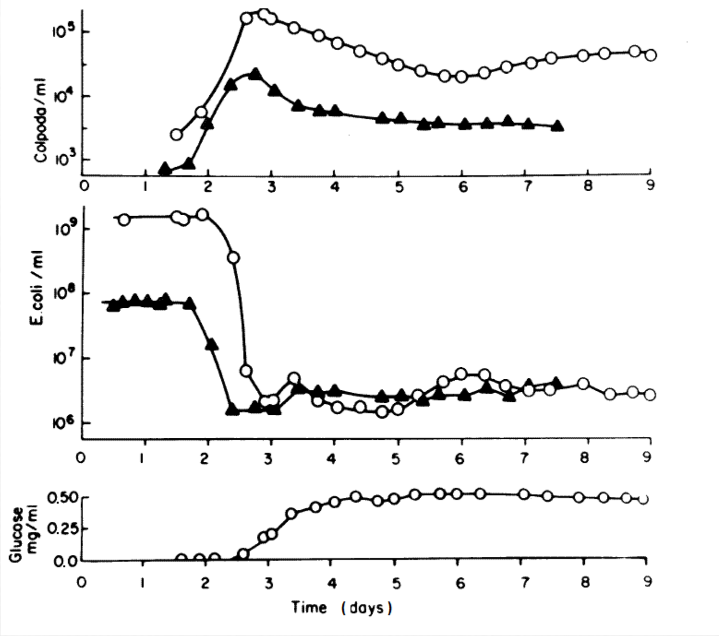
$$a = -\frac{d\bar{r}}{dN} = -\frac{\partial r}{\partial R}\frac{d\bar{R}}{dN} - \frac{\partial r}{\partial P}\frac{d\bar{P}}{dN} = a^{\mathrm{R}} + a^{\mathrm{P}}$$

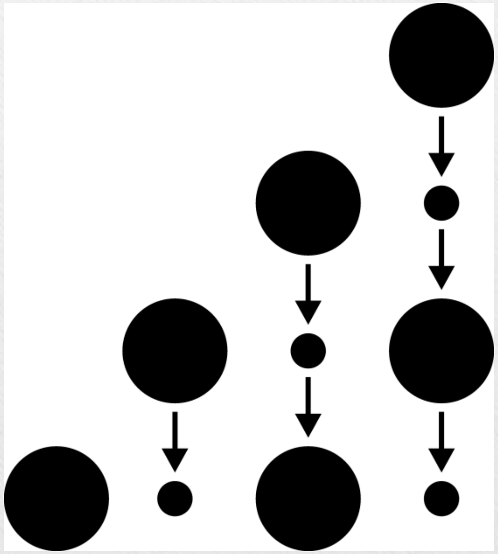
$$a^{\mathrm{P}} = -S^{\mathrm{P}} \cdot I^{\mathrm{P}} = \frac{\gamma^{\mathrm{P}}\left(\beta^{\mathrm{P}}\right)^2}{b} \to \infty \quad \text{when } b \to 0$$

Rule of exclusive resource limitation (Fretwell 1977)

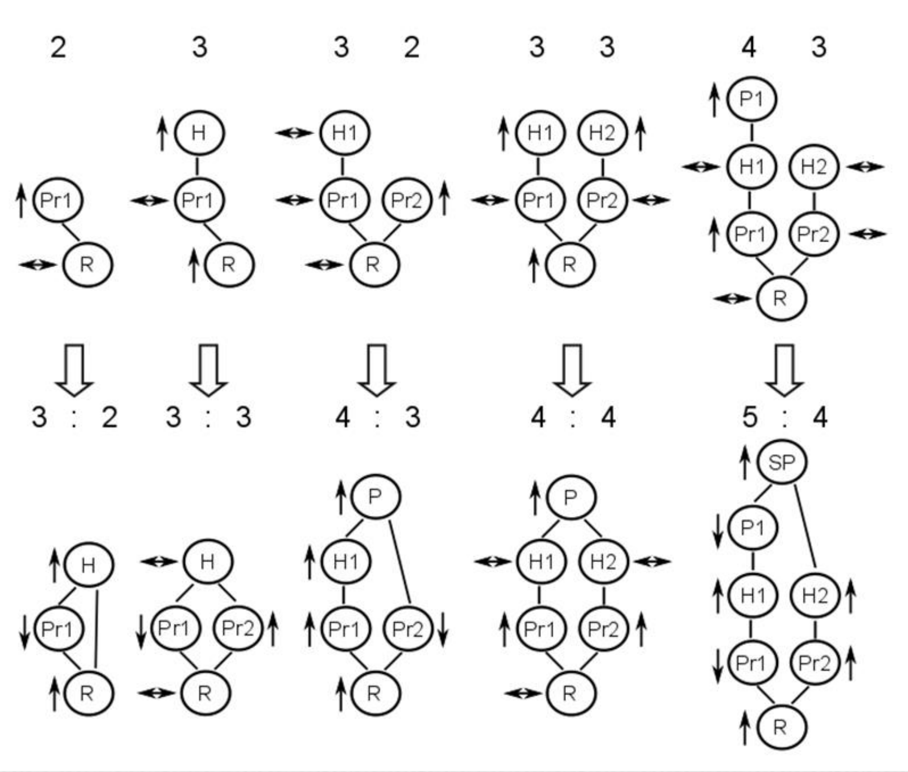
Exclusive resource limitation







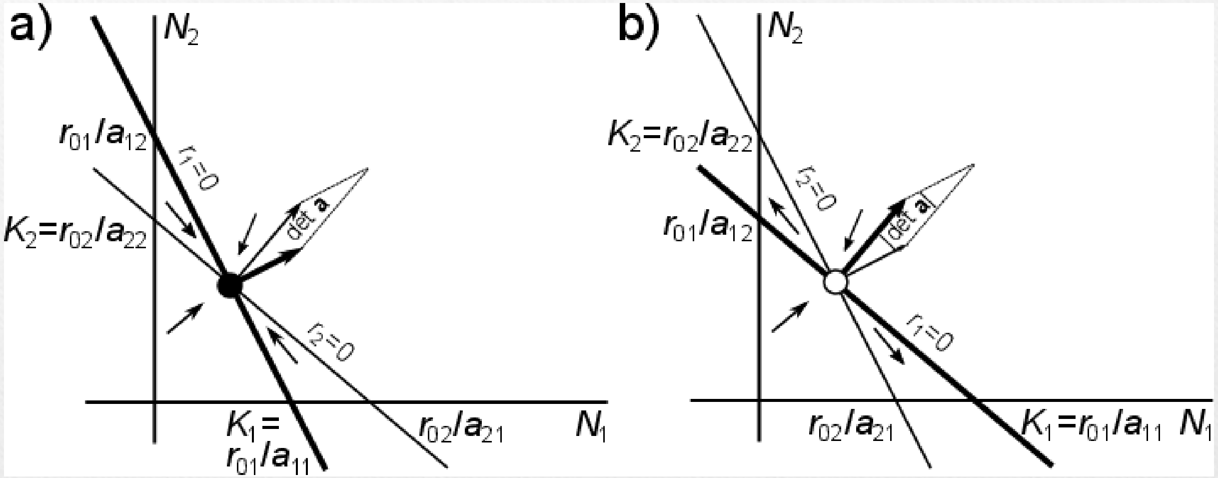
Consequence for trophic networks



How does the network react to an increase of resource input?

(Wallrab, Diehl, de Roos, 2012)

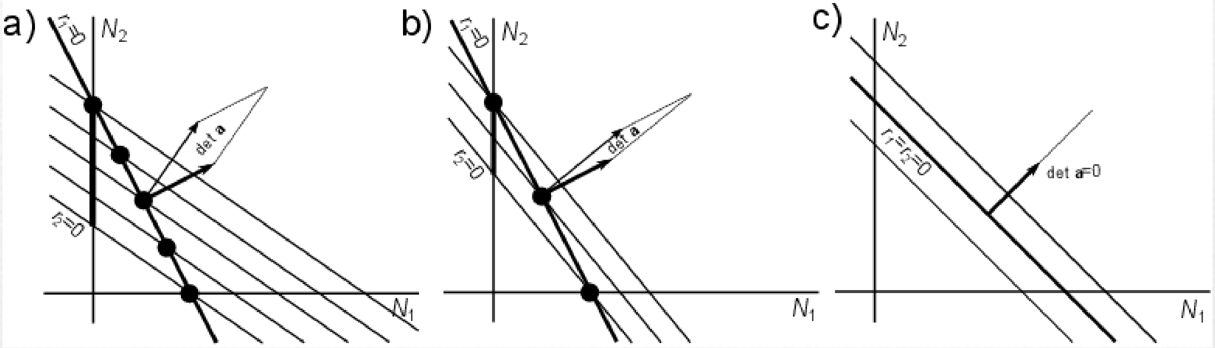
Coexistence in Lotka-Volterra



$$\frac{1}{N_1}\frac{dN_1}{dt} = r_{01} - a_{11}N_1 - a_{12}N_2$$

$$\frac{1}{N_2}\frac{dN_2}{dt} = r_{02} - a_{21}N_1 - a_{22}N_2$$

Robustness of coexistence



$\boldsymbol{r}(\boldsymbol{N}) = \boldsymbol{r}_0 - \boldsymbol{a}\boldsymbol{N} = 0$

$\boldsymbol{N} = \boldsymbol{a}^{-1}\boldsymbol{r}_0$

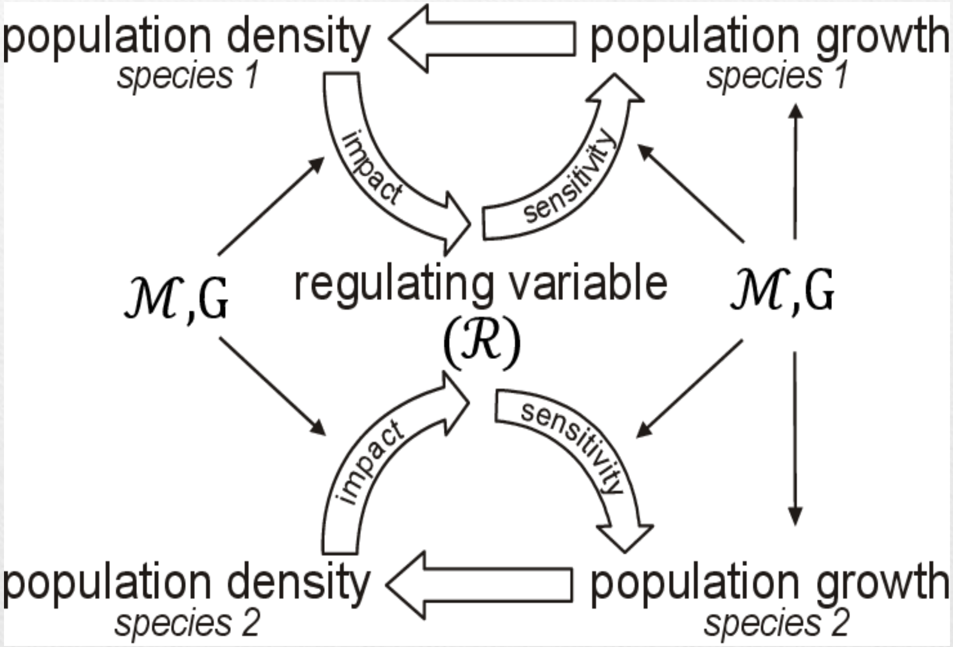
$$\boldsymbol{a}^{-1} = \frac{1}{\det \boldsymbol{a}}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

det $\boldsymbol{a}$ small

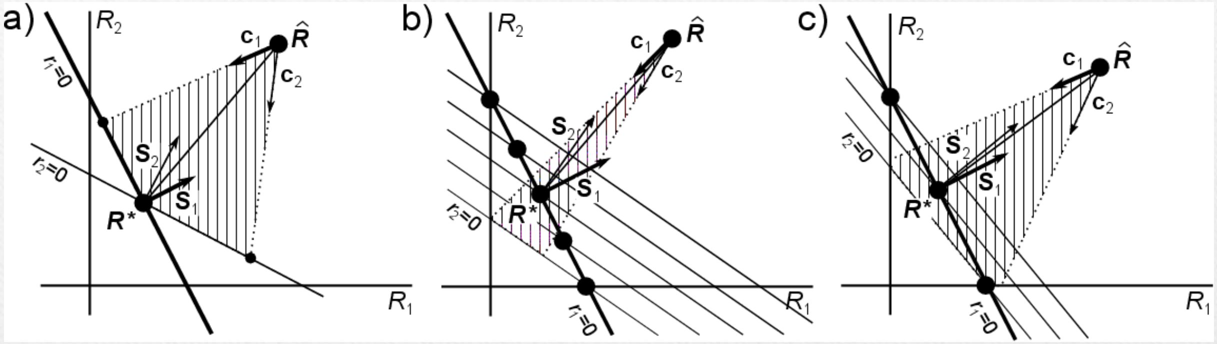
$\Downarrow$

Coexistence is not robust.

Interactions through regulating variables



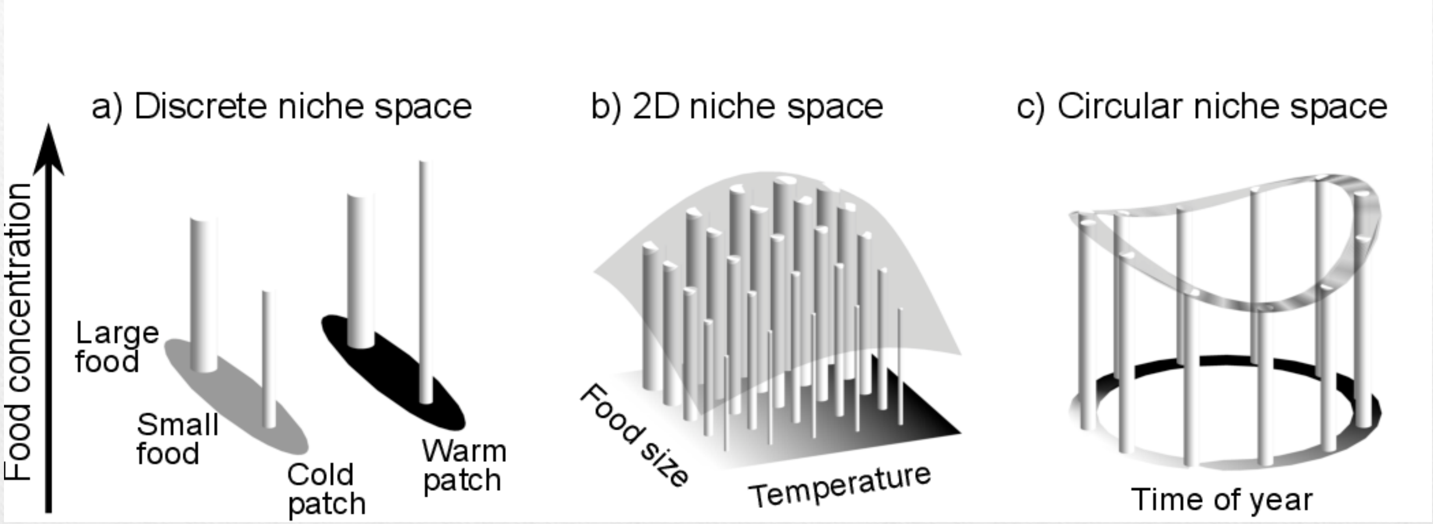
Robustness of coexistence



$$a_{ij} = -\frac{\partial r_i}{\partial N_j} = -\frac{\partial r_i}{\partial \boldsymbol{\mathcal{R}}} \cdot \frac{\partial \boldsymbol{\mathcal{R}}}{\partial N_j} = -\frac{\partial r_i}{\partial \mathcal{R}_1} \cdot \frac{\partial \mathcal{R}_1}{\partial N_j} - \frac{\partial r_i}{\partial \mathcal{R}_2} \cdot \frac{\partial \mathcal{R}_2}{\partial N_j} = -\boldsymbol{S}_i \cdot \boldsymbol{I}_j$$

Robust coexistence: Populations must differ sufficiently in both $\boldsymbol{I}$ and $\boldsymbol{S}$!

Niche space

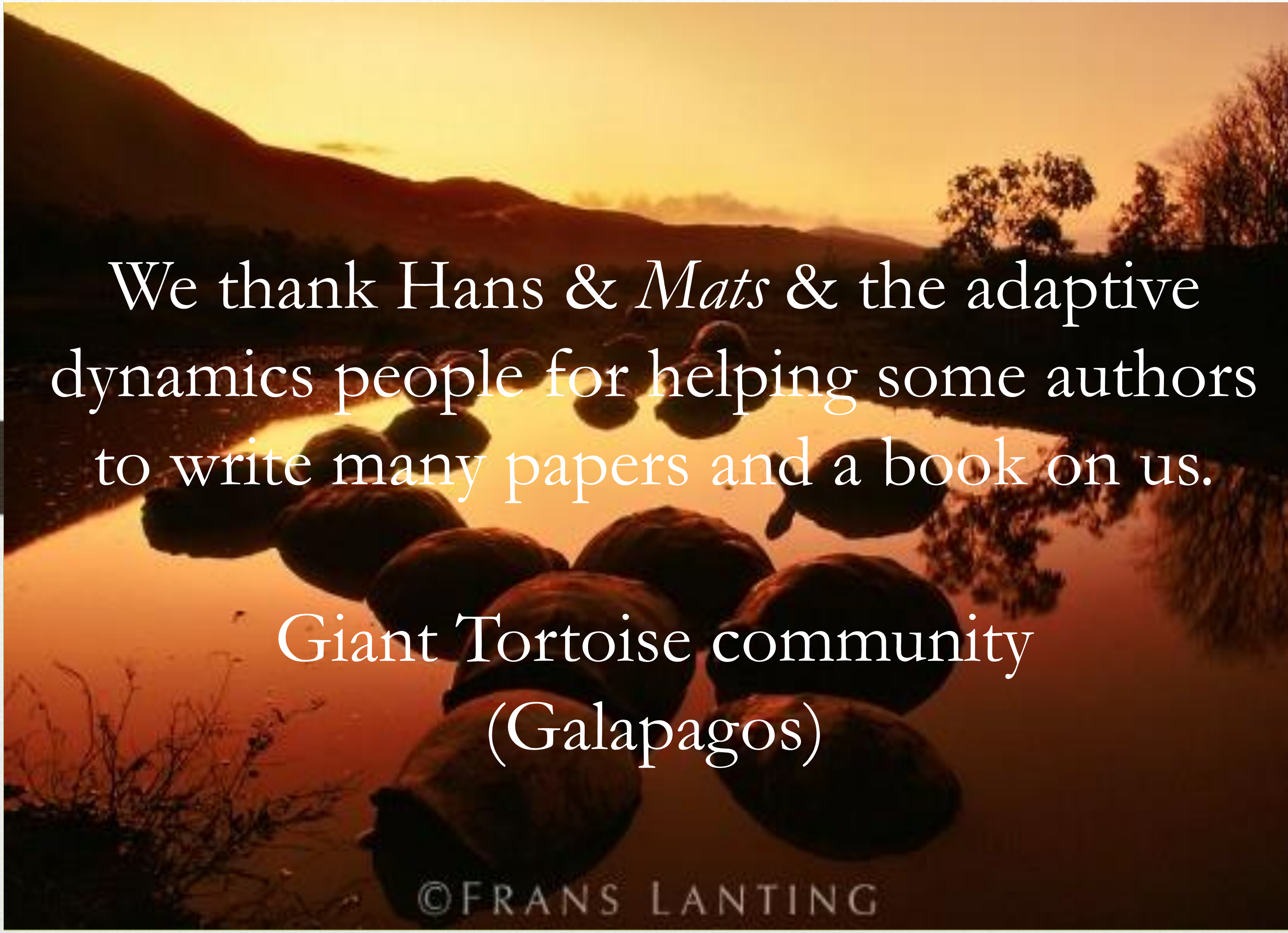


Niche space := index space of the regulating variables
≠ space spanned by the regulating variables

Niche space : discrete, or continuous

Mathematics of ecology $\subset$ Mathematics of life

- We suggest an integrated theory of ecology.
- It is tautologic, as any math; not a replacement for understanding your specific system..
- Without such integration, ecology would remain stuck in hopeless debates.
- We tried to keep math as minimalist, as possible in the book.
- Still, the integration would not be possible without deep mathematics.


We thank Hans & *Mats* & the adaptive dynamics people for helping some authors to write many papers and a book on us.
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