Géza Meszéna

Eötvös University, Budapest

Probability: From East to West Monash@Prato, 2017

◆□▶ ◆□▶ ◆□▶ ▲□▶ ▲□ ◆ ○ ◆ ○ ◆

Introduction





2 Niche theory

3 Models on fluctuation-induced coexistence

4 Conclusion

▲ロト ▲御 ト ▲ 臣 ト ▲ 臣 ト ● ○ ○ ○ ○

From East to West, my account



George Marx (1927-2002)

- Neutrino '72, Balatonfüred
- Zel'dovich meets West
- neutrino mass in cosmology
- birth of astroparticle physics

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Adaptatation to a fluctuation?



Liz Pásztor



Éva Kisdi

- How to adapt to fluctuation?
- Well, it's averaged out, anyway!
- Well, but it's complicated!
- Frequency dependence!

・ロト・日本・日本・日本・日本・日本

Adaptative dynamics



Hans Metz



Stefan Geritz

- Directional evolution
- Singular points
- Evolutionary branching

Speciation

Ecological niche



Mats Gyllenberg



Gyuri Barabás

- Niche segregation with respect to regulation
- No continuous coexistence
- Temporal niche segregation
- Evolutionary branching is a process of niche segregation

・ロト・日本・日本・ 日本・ シック・

Theory-Based Ecology: A Darwinian approach



Principles

- 1 Exponential growth
- 2 Growth regulation
- 3 Inherited variability
- 4 Finitness stochasticity
- 5 Competitive exclusion
- 6 Coexistence
- 7 Constraints trade-offs

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

Arbitrary, or adapted species?

There ara so many complications, why don't consider species, as random? Ideas in this vein:

No competitive exclusion in a fluctuating world

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

- Neutral theory of biodiversity
- Random interaction matrix
- Allopatric/genetic theory of speciation

My take: Adapted species!

Arbitrary, or adapted species?

There ara so many complications, why don't consider species, as random? Ideas in this vein:

No competitive exclusion in a fluctuating world

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

- Neutral theory of biodiversity
- Random interaction matrix
- Allopatric/genetic theory of speciation

My take: Adapted species!

Is there such thing, as niche theory?

Once upon a time we have it:

- Gause's principle & limiting similarity
- Lotka-Volterra model & resource utilization function
- Hutchinson's niche space
- Before long, the picture fell apart:
 - Ecology became too complicated for Lotka-Volterra.
 - Mechanistic models did not lead general results.
 - No clear conclusion on Gause and limiting similarity.
 - Attempts to interpret diversity via questioning competitition.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

- Since, theoretical ecology has grown up:
 - Beyond the specific models (eg. Caswell).
 - Renewed interest in coexistence theory (eg. Chesson).
 - Niche has remained controversial.

Is there such thing, as niche theory?

Once upon a time we have it:

- Gause's principle & limiting similarity
- Lotka-Volterra model & resource utilization function
- Hutchinson's niche space
- Before long, the picture fell apart:
 - Ecology became too complicated for Lotka-Volterra.
 - Mechanistic models did not lead general results.
 - No clear conclusion on Gause and limiting similarity.
 - Attempts to interpret diversity via questioning competitition.
- Since, theoretical ecology has grown up:
 - Beyond the specific models (eg. Caswell).
 - Renewed interest in coexistence theory (eg. Chesson).
 - Niche has remained controversial.

Is there such thing, as niche theory?

Once upon a time we have it:

- Gause's principle & limiting similarity
- Lotka-Volterra model & resource utilization function
- Hutchinson's niche space
- Before long, the picture fell apart:
 - Ecology became too complicated for Lotka-Volterra.
 - Mechanistic models did not lead general results.
 - No clear conclusion on Gause and limiting similarity.
 - Attempts to interpret diversity via questioning competitition.
- Since, theoretical ecology has grown up:
 - Beyond the specific models (eg. Caswell).
 - Renewed interest in coexistence theory (eg. Chesson).
 - Niche has remained controversial.

Is there such thing, as niche theory?

Once upon a time we have it:

- Gause's principle & limiting similarity
- Lotka-Volterra model & resource utilization function
- Hutchinson's niche space
- Before long, the picture fell apart:
 - Ecology became too complicated for Lotka-Volterra.
 - Mechanistic models did not lead general results.
 - No clear conclusion on Gause and limiting similarity.
 - Attempts to interpret diversity via questioning competitition.
- Since, theoretical ecology has grown up:
 - Beyond the specific models (eg. Caswell).
 - Renewed interest in coexistence theory (eg. Chesson).
 - Niche has remained controversial.

Steps from LV & classical niche to modern theory

Resources ⇒ Regulating variables

- 2 Lotka Volterra ⇒ linearization of dynamics
- 3 Resource utilization ⇒ impact & sensitivity
- 4 Limit of similarity ⇒ Robustness of coexistence



▲ロ ▶ ▲周 ▶ ▲ 国 ▶ ▲ 国 ▶ ● の Q @

Steps from LV & classical niche to modern theory

- Resources ⇒ Regulating variables
- 2 Lotka Volterra ⇒ linearization of dynamics
- 3 Resource utilization ⇒ impact & sensitivity
- 4 Limit of similarity ⇒
 Robustness of coexistence

Any model can be linearized! Lotka-Volterra competition:

$$r_i = r_{0i} - \sum_j a_{ij} n_j$$

Generalized competition coefficient:

$$a_{ij} = -\frac{\partial r_i}{\partial n_j}$$

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Steps from LV & classical niche to modern theory

- Resources ⇒
 Regulating variables
- 2 Lotka Volterra ⇒ linearization of dynamics
- 3 Resource utilization \Rightarrow impact & sensitivity
- 4 Limit of similarity ⇒
 Robustness of coexistence

Classical niche theory (ad hoc):

$$a_{ij} \sim \sum_{k} u_{ik} u_{jk}$$
rce utilization

Proposed theory (derived):

Resou

$$-a_{ij} = \frac{\partial r_i}{\partial n_j} = \sum_k \frac{\partial r_i}{\partial \mathcal{R}_k} \frac{\partial \mathcal{R}_k}{\partial n_j} = \mathbf{S}_i \cdot \mathbf{I}_j$$

Sensitivity of Species *i*
Impact of Species *j*

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Steps from LV & classical niche to modern theory

- Resources ⇒ Regulating variables
- 2 Lotka Volterra ⇒ linearization of dynamics
- 3 Resource utilization ⇒ impact & sensitivity
- 4 Limit of similarity ⇒
 Robustness of coexistence

Equilibrium:

$$r(\mathcal{R}(n), \mathcal{E}) = 0$$

Perturbation:

$$\frac{\partial \boldsymbol{n}}{\partial \boldsymbol{\mathcal{E}}} = \boldsymbol{a}^{-1} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{\mathcal{E}}}$$

Robustness:

 $\det \boldsymbol{a} = \det(\boldsymbol{S}_i \boldsymbol{I}_j)$

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

must be large! ⇒ Species should be different!

Steps from LV & classical niche to modern theory

- Resources ⇒
 Regulating variables
- 2 Lotka Volterra ⇒ linearization of dynamics
- 3 Resource utilization \Rightarrow impact & sensitivity
- 4 Limit of similarity ⇒
 Robustness of coexistence

Larger similarity in Impact or Sensitivity ↓ Weaker robustness of coexistence

No absolute limit of similarity!

Coexistence of a continuum is struturally unsable!

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Niche space: Ways of niche segregation

Varieties for niche space:



Niche space: set of regulating variables. Not necessarily an Euclidean space of a few dimension!

Complications abound

What about

- population structure?
- spatial structure?
- temporal structure?
- facilitation?
- etc.

We have papers about them...

▲ロ ▶ ▲周 ▶ ▲ 国 ▶ ▲ 国 ▶ ● の Q @

Models on fluctuation-induced coexistence

Disturbance-generated niche-segregation



Kalle Parvinen

- Coexistence by disturbation?
- Metapopulation with local catastrophes.
- Explicit population dynamics in the patches.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

AD origin of successional types.

Models on fluctuation-induced coexistence

Trade-off between fecundity and local competitivity



Models on fluctuation-induced coexistence

Species succession within a patch



Patch age t

Models on fluctuation-induced coexistence

Evolutionary branchings



 $\mathcal{O} \mathcal{O} \mathcal{O}$

Models on fluctuation-induced coexistence

Evolution, longer time-scale



Models on fluctuation-induced coexistence

Pairwise invasibiliy



◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

Models on fluctuation-induced coexistence

Fitness with dimporphic resident



▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

Models on fluctuation-induced coexistence

Trimporphic resident



Models on fluctuation-induced coexistence

Four residents



▲□▶▲圖▶▲≣▶▲≣▶ ■ のへで

Models on fluctuation-induced coexistence

Effect of relative nonlinearity



András Szilágyi

- Minimal model for fluctuation-mediated coexistence!
- Linear models average out!
- Let us see a qudratic one!

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

Models on fluctuation-induced coexistence

Model

Population dynamics affected by a white noise $\xi(t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}n_i(t) = \overbrace{\left[\sigma_i\xi(t) - a_i(n(t) - K_i) - b_i(n(t) - K_i)^2\right]}^{r_i(t)} n_i(t),$$

$$(n = \sum_i n_i)$$

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

Models on fluctuation-induced coexistence

Averaged equation

Average it out!

$$\overline{r}_i = -a_i(\overline{n} - K_i) - b_i(\overline{n} - K_i)^2 + \sigma_i \overline{\xi} = -a_i(\overline{n} - K_i) - b_i(\overline{n} - K_i)^2 - b_i V$$

◆□▶ ◆□▶ ◆□▶ ▲□▶ ▲□ ◆ ○ ◆ ○ ◆

We have two regulating variables:

- average \overline{n} of n
- variance V of n

At most two species can coexist!

But it was extreemly hard to see it in simulation...

Models on fluctuation-induced coexistence

Time series



Non realistic population sizes!

Models on fluctuation-induced coexistence

Time series, different scale



200

Models on fluctuation-induced coexistence

The REAL danger for extinction

Mean time to extinction in constant environment:

$$\mathrm{MTE} = C_1 e^{bK} \sim \infty$$

(Hamza, Jagers & Klebaner, 2015); same for fluctuating environment:

$$\mathrm{MTE}=\mathcal{C}_{2}\mathcal{K}^{c}\ll\infty.$$

(Ovaskainen & Meerson, 2010); exact theory?

Environmental & demographic stochasticity together is dangerous!

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・



Species have to adapt to *something*!

- They find their own constancy in the uncertain world.
- Coexisting species must differ in their way of regulation!
- There are no such things, as "equilibrium" vs. "fluctuating" ecology.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



- Species have to adapt to *something*!
- They find their own constancy in the uncertain world.
- Coexisting species must differ in their way of regulation!
- There are no such things, as "equilibrium" vs. "fluctuating" ecology.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



- Species have to adapt to *something*!
- They find their own constancy in the uncertain world.
- Coexisting species must differ in their way of regulation!
- There are no such things, as "equilibrium" vs. "fluctuating" ecology.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



- Species have to adapt to *something*!
- They find their own constancy in the uncertain world.
- Coexisting species must differ in their way of regulation!
- There are no such things, as "equilibrium" vs. "fluctuating" ecology.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで



- Species have to adapt to *something*!
- They find their own constancy in the uncertain world.
- Coexisting species must differ in their way of regulation!
- There are no such things, as "equilibrium" vs. "fluctuating" ecology.

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Thanks for the coworkers!

- György Barabás (University of Linköping)
- Zoltán Botta-Dukát (Institute of Ecology and Botany, Vácrátót)

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

- Mats Gyllenberg (University of Helsinki)
- Éva Kisdi (University of Helsinki)
- Hans Metz (University of Leiden)
- Kalle Parvinen (University of Turku)
- Liz Pásztor (Eötvös University, Budapest)
- András Szilágyi (Eötvös University, Budapest)

Thanks for your attention!